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DEFINITION AND USE OF THE PHI GRADE SCALE. (U)

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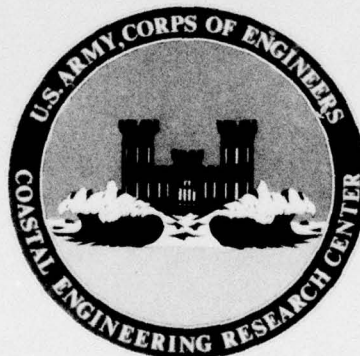
Definition and Use of the Phi Grade Scale

by
R.D. Hobson

COASTAL ENGINEERING TECHNICAL AID NO. 79-7
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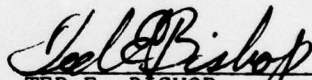
PREFACE

This technical aid provides an analysis of the phi grade scale used in describing sediment texture. The work was carried out under the sediment hydraulic interaction program of the U.S. Army Coastal Engineering Research Center (CERC).

The report was prepared by Dr. R.D. Hobson of the Engineering Geology Branch, Engineering Development Division, CERC.

Comments on this publication are invited.

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TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director

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CONTENTS

	Page
CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI)	5
SYMBOLS AND DEFINITIONS	6
I INTRODUCTION.	7
II GRADE SCALES.	7
1. Background	7
2. Common Classification Schemes	8
III PHI NOTATION.	8
1. Background	8
2. Phi Grade Scale	10
3. Terminology and Use	15
4. Conversions	16
IV EXAMPLE CALCULATIONS.	16
LITERATURE CITED.	18

TABLES

1 Grain-size scales--soil classification	9
2 Weight percentages by class of two typical beach sands	17
3 Phi sizes and millimeter equivalents of the phi mean for the grain-size distribution data shown in Table 2	17

FIGURES

1 Cumulative size-frequency plots comparing (a) millimeter and (b) phi-size scales	10
2 Size-frequency plots comparing (a) millimeter versus (b) phi-size scales	11
3 The normal curve (for $\mu = 2.0$, $\sigma = 0.70$).	13
4 Cumulative size frequencies on phi probability plot	14

CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI)
UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimete
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins ¹

¹To obtain Celsius (C) temperature readings from Fahrenheit (F) readings use formula: $C = (5/9) (F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9) (F - 32) + 273.15$.

SYMBOLS AND DEFINITIONS

$d(\text{mm})$	grain diameter expressed in millimeters
M_ϕ	phi mean of sample grain-size distribution (an estimate of μ)
Md_ϕ	50th percentile phi size
S_ϕ	phi sorting of sample grain-size distribution (an estimate of σ)
SK_ϕ	phi skewness measure of asymmetry for a sample grain-size distribution
ϕ	a measure of sedimentary particle size
μ	mean of a lognormal distribution
σ	standard deviation of a lognormal distribution

DEFINITION AND USE OF THE PHI GRADE SCALE

by
R.D. Hobson

I. INTRODUCTION

The Unified Soils, Wentworth, and phi grade scales are commonly used by coastal engineers to describe sediment texture. Of these, the phi scale is least understood. This report discusses why the phi scale was proposed initially, and how and when it should be used. Formulas and methods are presented for using the phi notation, calculating the mean grain size, and sorting of sediment samples, and for converting between phi- and millimeter-based size scales.

II. GRADE SCALES

1. Background.

Descriptive terms such as silt, sand, and gravel are used to describe natural sediments; e.g., silty sand indicates a dominantly sandy sediment containing some silt. These terms also imply actual particle-size ranges as defined by the particular classification scheme being used. The term, particle size, refers here to *grain diameter*, as determined by using standard sieving (Lambe, 1967) and settling techniques (Schlee, 1966).

Particle sizes vary on a continuous scale which is arbitrarily divided by a classification scheme into a convenient number of units for describing and analyzing sediments. These divisions or scale units are commonly called *grades*, which together constitute a *grade scale*. Each grade scale is arbitrary in the sense that it is created to reflect desired sediment properties or to facilitate the purpose for which it is used.

Most grade scales have unequal-size intervals which are advantageous for two main reasons. First, the sizes of natural sediments cover such a large range that an unwieldy number of equal-size grades are needed to classify them (e.g., a boulder 1 meter in diameter is 1 million times larger than a 1 micrometer-sized clay particle). Second, and more important, the unequal-size classes can be used to describe those differences that are important to the geologist or engineer. For example, a millimeter difference in boulder sizes is insignificant but the same difference between sand grain sizes is usually an important inequality.

Grade scales must be flexible enough to be used for analytic as well as descriptive purposes. Therefore, the most useful scales are usually those with grades that can be easily handled for computation purposes and with class limits that exhibit a systematic subdivision of particle sizes. *Geometric grade scales* are particularly advantageous where each subdivision (grade) bears a fixed ratio to preceding and succeeding grades. For example, particle sizes ranging from 1,000 to 0.01 millimeters could

be subdivided into five grades by the geometric series 1,000, 100, 10, 1, 0.1, and 0.01 millimeters where each grade limit in the series is one-tenth as large as the preceding one, or 10 times larger than the succeeding one.

2. Common Classification Schemes.

Udden (1898) introduced the first true geometric grade scale. He chose 1 millimeter as the starting point for his scale and used the ratio 1/2 (or 2) to create size classes with limits of 1/2, 1/4, 1/8 millimeter etc. (2, 4, 8 millimeters, etc.). Wentworth (1922) adopted and expanded Udden's geometric grade series, adding descriptive terms for the grades such as "sand" and "silt." He selected size limits for the grades that employed common usage of the terms by geologists and that reflected transport characteristics of different sediment sizes (e.g., clay sizes are commonly transported in suspension, whereas sand is usually rolled or saltated along the bed). The resulting Udden-Wentworth grade scale (called the *Wentworth Classification*, Table 1) is generally preferred by geologists. It is geometric with fixed ratio 2, and consists of 24 classes that systematically span the range from 1/4096 to 4,096 millimeters. The width of each class relates directly to the diameters of grains within it so that coarse grains are described in terms of classes with relatively wide ranges of size, and fine particles by classes of fairly narrow width.

The *Unified Soils Classification* (Table 1) is the most common grade scale used by soil scientists and engineers. This scale was developed by Casagrande (1948), adopted by the Corps of Engineers (U.S. Army Engineer Waterways Experiment Station, 1953) and the American Society for Testing Materials (ASTM), and is based on the mesh size of sieves used for the mechanical analysis of sediments. The Unified scale is also geometric because sieve openings are graduated at the fixed ratio $\sqrt[4]{2}$ (or 1.1892) and, starting at 4 millimeters, every fourth value in the scale agrees with the Wentworth class limits.

Table 1 has been constructed to show how grade limits and descriptive terms compare for the Unified Soils and Wentworth classification schemes. Although generally similar, the two schemes do assign somewhat different size ranges (in millimeters) to each sediment category. For example, the total range of sand sizes in the Unified Soils scheme is 0.074 to 4.76 millimeters as opposed to 0.062 to 2 millimeters for the Wentworth. Because of these differences, communication problems with terms can be encountered and care must be taken to identify the classification scheme being used.

III. PHI NOTATION

1. Background.

Geometric grade scales are not necessarily best for all types of sediment-size analysis. Although the property of fixed-size ratio among

Table 1. Grain-size scales--soil classification (modified from U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977).

Unified Soils Classification		ASTM Mesh	mm Size	Phi Value	Wentworth Classification	
COBBLE			256.0	-8.0		BOULDER
			76.0	-6.25		COBBLE
COARSE GRAVEL			64.0	-6.0		
			19.0	-4.25		
FINE GRAVEL			4	4.76	-2.25	
			5	4.0	-2.0	
SAND	coarse		10	2.0	-1.0	
			18	1.0	0.0	
	medium		25	0.5	1.0	
			40	0.42	1.25	
	fine		60	0.25	2.0	
			120	0.125	3.0	
			200	0.074	3.75	
			230	0.062	4.0	
				0.0039	8.0	
				0.0024	12.0	
SILT						SILT
CLAY						CLAY
						COLLOID

classes produces a systematic and logical division of particle sizes, this same property can also create some unique problems for the statistical analysis and graphing of size data. In statistics, sample size often affects analysis results; therefore, it is desirable to have a size scale with class limits that can be easily halved or quartered in order to provide an adequate number of experimentally determined points for analytic purposes. Geometric scales can be subdivided into smaller equal-sized classes but the class limits produced are often irrational rather than of integer value and more difficult to handle quantitatively. An arithmetic-size scale would be easy to subdivide and could be derived from an existing geometric scale through the use of an appropriate logarithmic transformation.

Graphing techniques are commonly used for comparing the grain-size distributions (gsd) of different sediment samples. Plots of cumulative proportion (usually weight percent) of sediment coarser than a series of size classes tend to be fairly straight and steep in the less than 1-millimeter class size, and then to "tail out" toward the coarser sizes. The shapes of plots for different sample gsd's might appear similar even though there are important textural differences. If the differences occur in the finer sizes, this kind of diagram tends to push these sizes together rather than to accentuate them (Fig. 1,a). This graphing problem, like the statistical problem above, could also be solved by using logarithms to transform the geometric-size scale into an arithmetic scale.

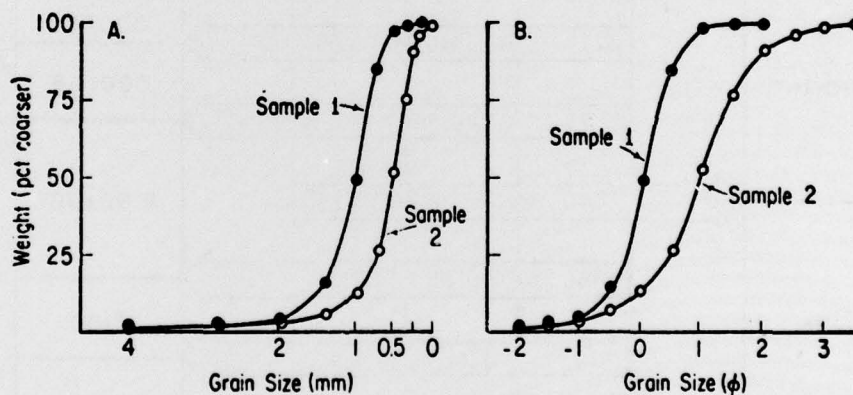


Figure 1. Cumulative size-frequency plots comparing (a) millimeter and (b) phi-size scales.

2. Phi Grade Scale.

The phi notation, introduced by Krumbein (1934, 1938), is used to transform the geometric Wentworth scale into an arithmetic scale where

$$\phi = -\log_2 (d(\text{mm})/1\text{mm}) , \quad (1)$$

and $d(\text{mm})$ is the grain diameter in millimeters. This transformation uses the logarithm to the base 2, which is equal to the power of the geometric series, and produces a *dimensionless, arithmetic-size scale* that can be easily divided into smaller units with limits of integer value. Differences in the shapes of the gsd's using the phi-size scale can be seen by comparing a and b in Figure 1 in which the range of finer grain sizes has been significantly expanded. Also, the plots of weight percent for each size class tends to be fairly symmetric about the most frequently occurring sizes when phi is used (Fig. 2, a versus b).

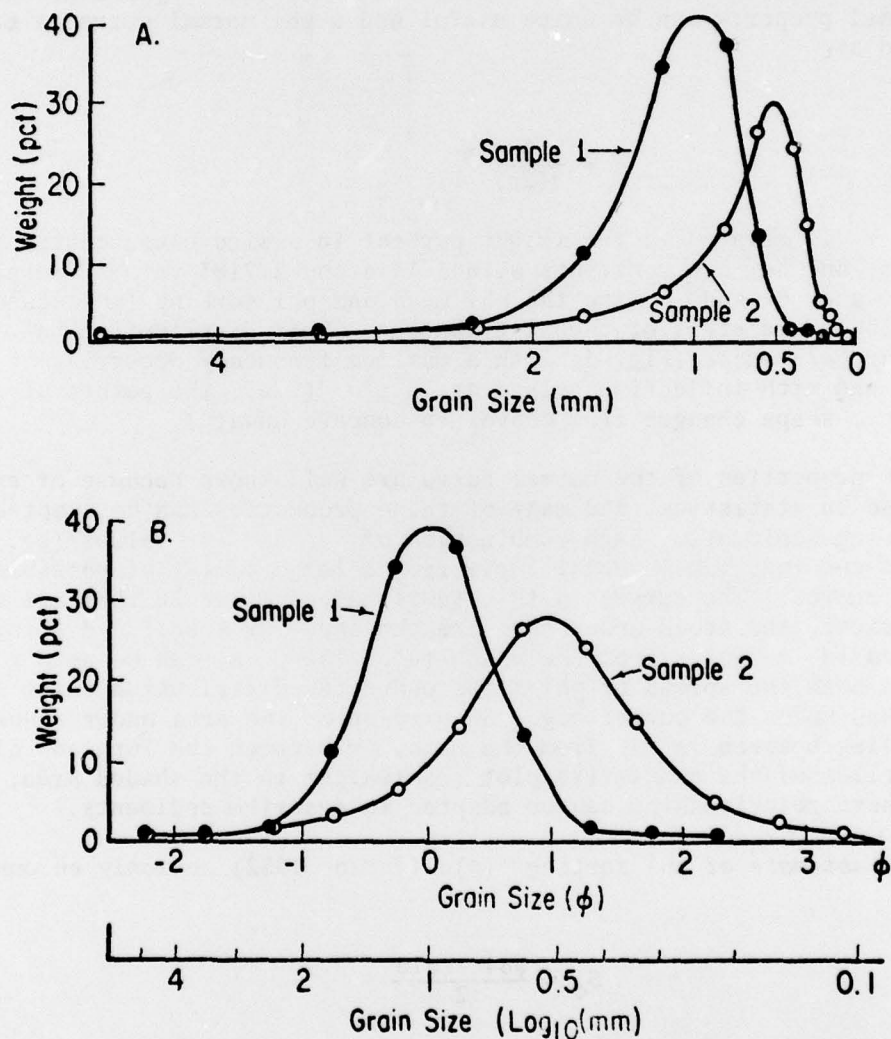


Figure 2. Size-frequency plots comparing (a) millimeter versus (b) phi-size scales.

In equation (1), ϕ is the transformed ratio of lengths with 1 millimeter serving as the standard diameter for comparison purposes (i.e., when $\phi = 0$, $d = 1$ mm). Because ϕ is dimensionless, it should not be used in circumstances where a length dimension is required (e.g., in a Reynolds number). Also, the negative sign in equation (1) has the effect of giving a positive ϕ value to finer sizes and negative ϕ 's to coarse sizes. This is reasonable since most natural sediments fall within the finer (positive ϕ) size grades, but it takes time to become familiar with ϕ terms where decreases in ϕ value indicate increases in actual grain size. Despite these minor problems the logarithmic ϕ transformation has the effect of changing the plot of many sediment distributions into the shape of essentially normal distributions: hence, the millimeter-size distribution is sometimes called *lognormal*. This lognormal property can be quite useful and a ϕ normal curve is expressed as:

$$Y = \frac{1}{\sigma(2\pi)} e^{-\left[\frac{(\phi-\mu)^2}{2\sigma^2}\right]} \quad (2)$$

where Y is related to the weight percent in a size class containing ϕ , π and e are constants with 3.1416 and 2.7183 values, respectively, and μ and σ are the ϕ mean and ϕ sorting (ϕ standard deviation) parameters of the distribution. This distribution has the familiar *bell shape* (Fig. 3) with a maximum frequency occurring at $\phi = \mu$ and with inflection points at $\mu \pm \sigma$ (i.e., the points at which the curve shape changes from convex to concave upward).

The properties of the normal curve are well known because of extensive use in statistics, and many of these properties can be adapted for describing sediments. Each combination of μ and σ values (eq. 2) defines one individual normal curve from a large family of possible normal curves. The curves in this family are similar in that all are symmetrical, and areas under each are the same for specific distances measured in σ units from the mean (μ). Thus, σ can be used to measure both the spread of ϕ sizes under the distribution curve and the areas under the curve; e.g., 68 percent of the area under a normal curve lies between $\pm 1\sigma$ from the mean, or between the 16th and 84th percentiles of the cumulative plot (equivalent to the shaded area, Fig. 3). These relationships can be adapted to describe sediments.

One *estimate* of ϕ sorting (σ) (Inman, 1952) commonly encountered is:

$$S_{\phi} = \frac{\phi_{84} - \phi_{16}}{2} \quad (3)$$

If an actual distribution were completely symmetrical, the mean (μ) would be located at the 50th percentile ϕ size (ϕ_{50}) or be equal to the median size (Md_{ϕ}). However, it is common practice to select the

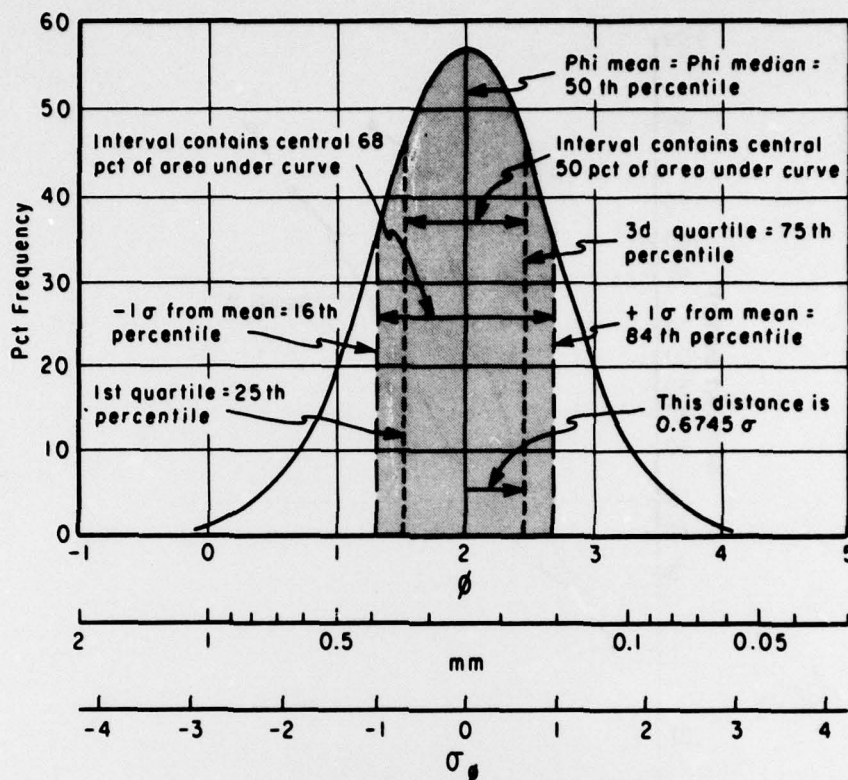


Figure 3. The normal curve (for $\mu = 2.0$, $\sigma = 0.70$). Shaded area for interval $\pm 1\sigma$ from mean (μ) contains the central 68 percent of the area under the curve (adapted from Krumbein, 1957).

following estimate of the mean which is statistically more efficient and less biased than the median for cases where the actual gsd is not completely symmetrical.

$$M_{\phi} = \frac{\phi_{84} + \phi_{16}}{2} \quad (4)$$

For a symmetrical distribution, equation (4) will produce the same value as the median. S_{ϕ} and M_{ϕ} (eqs. 3 and 4) are probably the best estimates of σ and μ (Inman, 1952) for describing unimodal sedimentary grain-size distributions.

A common way to obtain these parameters is by using a graphical technique (Fig. 4). The sample size data are plotted as a cumulative distribution on log (phi) probability paper. This paper is constructed so that a lognormal distribution will plot as a straight line. The plots of sample distributions that are asymmetric will not be straight. The degree of asymmetry, or nonnormality, can be determined by comparing the observed distribution with a straight "approximation" curve drawn

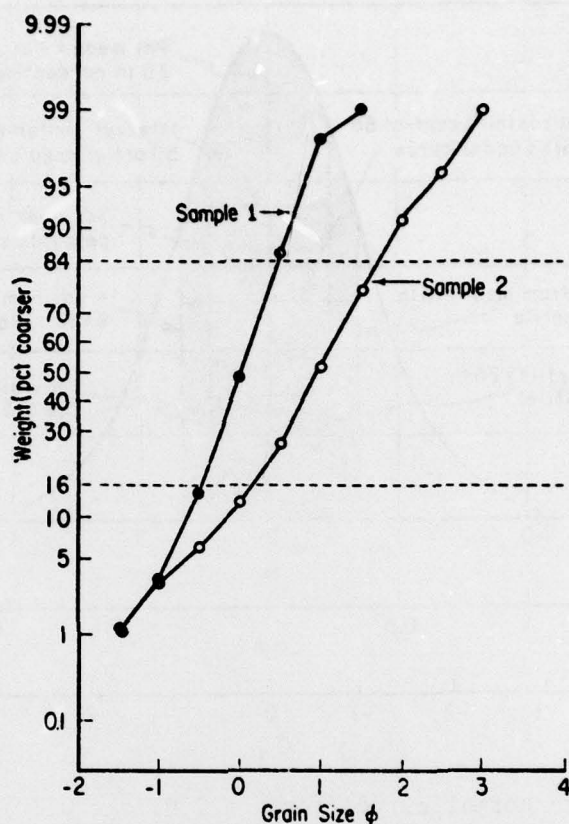


Figure 4. Cumulative size frequencies on phi probability plot (data from Table 2).

through the 84th and 16th percentile intercepts of the observed curve. The comparison can either be made qualitatively by noting the size of the "gap" between the curves along the phi size equal to the mean, or quantitatively by computing an estimate of the skewness parameter.

$$SK_{\phi} = \frac{(M_{\phi} - Md_{\phi})}{S_{\phi}} \quad (5)$$

In both cases, the difference between the mean and median sizes is reflected by the observed asymmetry. For example, a negative skewness exists when the observed distribution lengthens or tails out toward the coarser, negative phi sizes. In this case, the mean (center of gravity) is more affected by the long, coarse tail than by the position of the median. Positive skewness arises when the curve tails toward the finer, positive phi sizes.

Skewness differences among sediment samples are frequently used to compare sediment-size distributions to characterize sedimentary

environments and analyze the response of sediments to varied flow conditions. These comparisons can be quite effective, especially when the parameter is used within some multivariate analysis scheme. However, the skewness parameter is not as stable statistically as the mean and sorting parameters and small deviations from normality can result in fairly large skewness variations.

3. Terminology and Use.

The phi scale is less familiar to engineers than to geologists and its use has traditionally created some problems. Many of these problems arise from improper use of terminology and from incorrect conversions between phi and geometric grade scales. Although millimeter equivalents can be assigned to individual phi values, the phi notation is dimensionless. The symbol " ϕ " represents a ratio of lengths (eq. 1) and identifies the *origin* of the value it follows. It does not have the same significance as the dimensional abbreviation "mm" which indicates *in what units* the measurements were made. McManus (1963) suggested that one way to keep the meaning of these symbols straight is to place ϕ only after values that indicate a single particle size (e.g., $M_\phi = 3.0\phi$, or diameter = 2.0ϕ), and to use the notation "phi unit" following an interval value such as sorting (e.g., $S_\phi = 2.5$ phi units). Thus, sorting as defined by equation (2) is the interval on a graph representing the number of Wentworth grades occurring on either side of M_ϕ as defined by the concept of standard deviation (e.g., 1 phi unit = 1 Wentworth grade). Finally, since sorting values are the number of phi units, they cannot be converted directly into millimeter value. Sorting values in millimeters can be calculated directly using appropriate formulas or, if desired, the phi values at $M_\phi \pm 1 S_\phi$ can be converted to millimeters.

Although no single grade scale will best serve all uses for describing texture, the phi scale does have the following advantages as summarized by the Inter-Society Grain Size Committee of the Society of Economic Paleontologists and Mineralogists (from Tanner, 1969):

- (a) Evenly spaced division points, facilitating plotting;
- (b) geometric basis allowing equally close inspection of all parts of the size spectrum;
- (c) simplicity of subdivision of classes to any precision desired, with no awkward numbers;
- (d) wide range of values, extending automatically to any extreme;
- (e) widespread acceptance;
- (f) coincidence of major dividing points with natural class boundaries (approximately);

- (g) ease of use in probability analysis;
- (h) ease of use in computing statistical parameters;
- (i) amenability of more advanced analytical methods;
- (j) fairly close approximation to most other scales, allowing easy adoption; and
- (k) phi-size screens are available commercially.

No other grade scale is even close to satisfying this list and few have more than three or four of these advantages.

4. Conversions.

Krumbein (1957) and U.S. Army, Corps of Engineers, Coastal Engineering Research Center (1977) provide a table for converting millimeters to phi units. Conversions between phi units and millimeters can also be performed easily on pocket calculators using the following equations:

$$\phi = -1.4427 \log_e (d(\text{mm})/1\text{mm}) \quad (6)$$

$$d(\text{mm}) = (1\text{mm}) (2^{-\phi}) \quad (7)$$

IV. EXAMPLE CALCULATIONS

Table 2 gives the weight percentages for the two sample gsd's shown in Figures 1 and 2. These textural data are typical for beach sands taken from the swash zone (sample 1) and the upper foreshore (sample 2) and then shaken through a nest of wire-mesh sieves that are size-graded at 0.5-phi intervals. Figure 4 shows these same data replotted on log probability paper. Table 3 contains the phi values at the 16th and 84th percentiles (ϕ_{16} and ϕ_{84}), the phi mean (M_ϕ) and phi sorting (S_ϕ) values as calculated using equations (3) and (4). The millimeter equivalents for the phi means are also included.

Inspection of Figure 4 reveals that the samples are essentially log-normal as evidenced by their fairly straight-line plots through the central region of the graph and confirmed by the symmetrical bell shapes shown in Figure 2(b). Also, the slopes of the gsd's can be used to quickly evaluate sorting differences. Equal sloping plots have the same sorting; however, steeper plots, such as for sample 1, indicate better sorted sediment (smaller S_ϕ) than for flatter ones like sample 2 (e.g., 0.48 versus 0.81 phi units, Table 3). Finally, it is reemphasized that phi means can be directly assigned equivalent millimeter values (as in Table 3) using appropriate tables or equation (7), but that phi sorting represents the number of Wentworth grades on each side of the phi mean and thus cannot be directly assigned a millimeter value.

Table 2. Weight percentages by class of two typical beach sands.

Mesh No. ¹	Mesh size (mm)	Phi value	Weight percent sample	
			1	2
5	4.00	-2.0	0.0	0.0
7	2.83	-1.5	1.0	1.0
10	2.00	-1.0	2.0	2.0
14	1.41	-0.5	11.0	3.0
18	1.00	0.0	34.0	6.0
25	0.71	0.5	37.0	14.0
35	0.50	1.0	13.0	26.0
45	0.35	1.5	1.0	24.0
60	0.25	2.0	1.0	15.0
80	0.18	2.5	0.0	5.0
120	0.13	3.0	---	3.0
170	0.09	3.5	---	1.0
230	0.06	4.0	---	0.0

¹Mesh numbers are ASTM-assigned numbers for sieves with openings equal to the millimeter mesh size shown.

Table 3. Phi sizes and millimeter equivalents of the phi mean for the grain-size distribution data shown in Table 2 (presented in manner suggested by McManus (1963) and as discussed in Section III.3).

Sample (No.)	$\phi 16$	$\phi 84$	Mean		S_{ϕ}
			ϕ	(mm)	
1	-0.48 ϕ	0.48 ϕ	0.00 ϕ	1.00	0.48 phi units
2	0.14 ϕ	1.76 ϕ	0.95 ϕ	0.52	0.81 phi units

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